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RESONANCE SCRAPING

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Protons lost in a ring leave at a few preferred locations, determined by some non-linear property the dipoles. This paper suggests taking control of lost protons by beating the magnets at their own game - by means of a *designed resonance* used as a beam scraper. It is a study of suitable resonances, including estimates of the required multipole element strengths. The appropriate resonances are *two-dimensional* and not much has been written about them because of their four-dimensioned phase space. A large number of figures is included to help penetrate the mysteries.

Protons "lost" from a beam in a high energy ring tend to strike the vacuum wall at a few preferred places, presumably as a result of the pattern of non-linear field at extreme amplitudes. This process is not well understood and the preferred locations cannot be controlled. It does depnd on machine parameters and the locations are unpredictable for major changes. For superconducting rings this concentration of losses greatly increases the probability of magnet quenches on small beam loss; for colliding beams experiments it may create an intolerable background if one of the locations is near a low-beta region.

In low energy rings *beam scraping* -pushing the beam slowly against an internal "target"- has provided a (somewhat) controled way of disposing of unwanted, large amplitude beam. In high energy rings scraping is difficult because the small, penetrating beam traverses only the salient edge of the target which acts as a beam scatterer instead of a beam stop and losses at the preferred places are enhanced. A good scraper would be most useful, for example one could create narrow beams for probing field problems, as well as providing quench and background protection.

The basic idea in this paper is to beat the beam to the punch by providing a ***designed resonance*** which is effective at much less than extreme amplitudes and which directs the "lost" protons to our own carefully designed "preferred location". The actual scraper then is the resonance ***separatrix***, which is very sharp and has no radiation thickness.

This paper investigates the properties of some resonances that could well serve as practical scrapers, in general *two-dimensional* resonances. The choice of resonance depends on details of space availability in the lattice and is perhaps impossible within the constraints of the Doubler, but new rings could easily incorporate a separatrix scraper into the general process of beam disposal.

The Basic Idea

We describe beams in *beta space* where all amplitudes and displacements use a reference β_0 (100 m. for the Doubler). To obtain real displacements one must multiply by $(\beta/\beta_0)^{1/2}$. [This is also the "conversational" space for rings - if I say that the amplitudes are 8 and 10 mm for horizontal and vertical, then one expects maximum displacements of 8 and 10 mm in the arcs, but of course one will be in the F quads and the other in the D's. It would be confusing to say 8 and 5 mm in the F's, which is the same thing.]

In beta space the equations for linear motion are simple

$$x = a \cos \varphi \qquad y = b \cos \vartheta$$
$$x' = - a \sin \varphi \qquad y' = - b \sin \vartheta$$

and these define my terminology.

Proton beams are suprisingly *gaussian* and much the same size in horizontal and vertical, that is the beam density measured as a function of x and measured as a function of y are both *normal* distributions which have the same σ when adjusted to β_o. The amplitude $a = (x^2 + x'^2)^{1/2}$ is a *circular normal (Rayleigh)* distribution with zero for $a = 0$ and a maximum at σ. The combined distribution for a and b is shown in figure 1. The important point is that large a with small b is not common, or *vice-versa*, but $a = b$ is important. Any multiple scattering process, like gas scattering, increases σ. Large but rare single scatterings produce a wide, thin pedestal which must have much the same type of distribution.

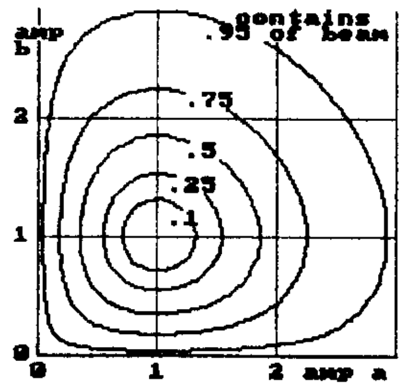


figure 1.

Figure 2 shows how one might apportion the linear betatron space to the various stages of resonant scraping. The high energy beam in the lower corner extends about 2mm ($\sigma = 2/3mm$). The line from 6mm to 6mm is an *adiabatic separatrix* from one of the later examples. The position can be adjusted downward by tuning closer to the resonance. This line is the stability limit for slow tuning (or slow emittance growth). Protons beyond the limit leave along the sloping trajectories. Note how all particles approach the same ratio of a to b. The growth per turn increases rapidly. A proton that just misses the target will strike at the dotted line (after ~7 turns), about 4mm into the target. Just as in *extraction*, it takes a lot of "good" aperture to develop a good separatrix scaper.

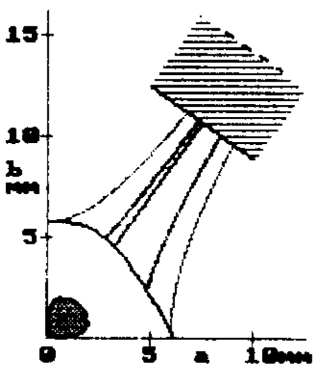


fig. 2 Scraping

This device can operate in two ways. First one can, at regular intervals, remove halo by tuning towards the resonance and then away. It is presumed that this periodic cleaning would inhibit background but this requires such a special set of growth times that it is questionable. (Sharpening a beam at low energy would be done this way.)

A much better mode of operation would be to leave the separatrix in an intermediate position, as in the diagram, and to use a resonance which has very little effect at beam amplitudes. This mode provides protection against any moderately slow (msec) loss. The beam-beam interaction is very short range and will have no effect at the separatrix, for example the infamous tune-shift applies to <u>small</u> amplitudes and practically vanishes at 3σ.

The similarity to extraction is striking but there are important differences. Extraction primarily selects particles on tune variation with momentum (chromaticity), whereas we want to select on betatron amplitude with the chromaticity zero (for amplitudes near the separatrix). Furthermore extraction does not much care about effective emittance dilution for particles not yet extracted (it might help), but we most certainly do. Finally we must scrape in both dimensions, which somewhat complicates the expanding trajectories and would be an unnecessary complication for extraction.

The Resonances

Figure 3 is a *tune plot* showing the resonances that we will consider. The dashed lines are resonances driven by skew multipoles and they are identical to their normal counterparts with ν_x and ν_y interchanged (also **a, b** etc.). The label 3,2 indicates $3\nu_x + 2\nu_y = integer$. The label 4-1 indicates $4\nu_x - \nu_y = int.$, an octupole non-linear coupling. One resonance which would be ideal, the octupole $2\nu_x + 2\nu_y = int.$ has been omitted because this is the only resonance guaranteed to affect beam-beam interaction. Operation near 1/4 is prohibited. We will also ignore the one-dimensional resonances ($3\nu_x$ and $5\nu_x$). For these resonances to scrape all large amplitude protons would require coupling horizontal and vertical so that amplitudes exchange slowly but almost completely. This requires a weak coupling and tuning very close to $\nu_x = \nu_y$, which is well into the "circle of confusion" where many resonances coincide. This is not a reliable operating condition and is probably a bad tune from beam-beam considerations.

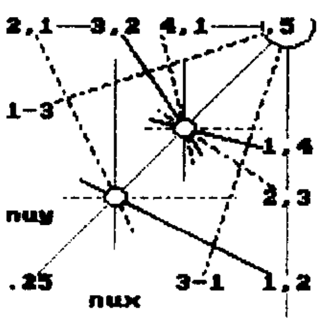


Fig. 3 Tune Plot

These resonances are 4-dimensioned creatures which makes their proprties difficult to see and we will rely on diagrams for comparisons, but summaries of the equations are included for more serious study. You may want to consult the companion paper ***Resonances and Resonance Widths*** for a better introduction. First we must define some terms.

We will use *sextupole* and *decapole* magnets. The strengths are

$$(Sl),\ B_y = S\ (x^2 \ldots \qquad \text{and} \qquad (Dl),\ B_y = D\ (x^4 \ldots$$

The resonances are $m\nu_x + n\nu_y = \mathit{integer} + (m^2 + n^2)^{1/2}\,\delta$

where δ is the perpendicular distance on a tune plot from the resonance line. There is a combination phase angle for each resonance

$$\alpha = m\varphi + n\vartheta \quad \text{with } \delta_\alpha = 2\pi(m^2 + n^2)^{1/2}\delta \quad \text{extra phase per turn.}$$

Resonances are examined turn-by turn. Non-linear effects average out except for the resonant component which accumulates for many turns because δ_α is small. To make this componenet effective we must use a pattern of multipole magnets *(Ml)* which accentuates the *driving terms*, sums for one turn

$$A = (\beta_0/(B\rho))\ \textstyle\sum(h^m v^n (Ml)_s \cos\alpha_s \qquad B = \ldots\ldots\sin\alpha_s$$
$$h = (\beta_x/\beta_0)^{1/2} \qquad v = (\beta_y/\beta_0)^{1/2}$$

Usually we choose our "observation point" so that $B = 0$, *A pos.* to simplify the expressions (this is only a phase shift in α and has no other effect). Note that the driving terms are *dimensioned*, we will use *(per cm.)* for *A* from sextupoles and *(per cm³)* for *A* from decapoles. [Thirds resonances arising from decapoles have a different dependence on *h* and *v*. These forgotten resonances are strong and provide an important scraper. Details below.]

With the aid of the driving term one can express the motion in small changes per turn (differentials) as a function *a*, *b*, α, and δ_α at the start of the turn. Before doing so however it will be convenient to express *a*, *b* in a scaling unit a_0 which is chosen to make the resonance diagrams easy to compare. These units are of the form

$$a_0 = c\ \delta/A\ \ (\text{sex.}) \quad \text{or} \quad a_0^3 = c\ \delta/A\ \ (\text{dec.})$$

where *c* is a constant. (a_0 will subsequently be set near 1 cm.)

We can always combine the equations *da/dN* and *db/dN* to find

$$n\,a^2 + m\,b^2 = F \quad \text{the "family" constant.}$$

These are hyperbolas on an *a–b* plot. A proton under the influence of this resonance never leaves its family line. We use this to analyze the resonance properties family by family. Each family is effectively one-dimensional.

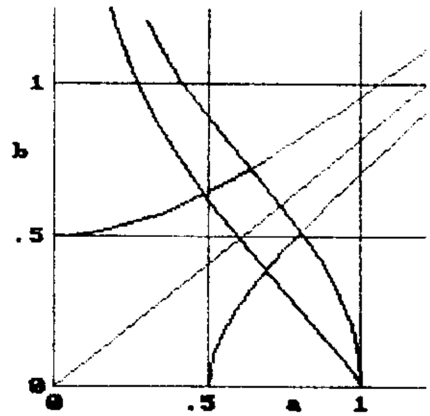
There is a line of *fixed points*, combinations of *a*, *b*, (and $\alpha = 180°$) for which *a*, *b*, and α do not change. We can also construct a *constant of the motion* which gives trajectories in phase space. The most important is the trajectory that contains the fixed point, the *separatrix*. Fixed points and separatrices vary from family to family.

$3\nu_x + 2\nu_y = \text{int} + \delta\sqrt{13}$ decapole

$A = (\beta_0/B\rho)\ \sum(h^3v^2(Dl))_s \cos\alpha_s$
$a_0^3 = 2\delta_\alpha/A$

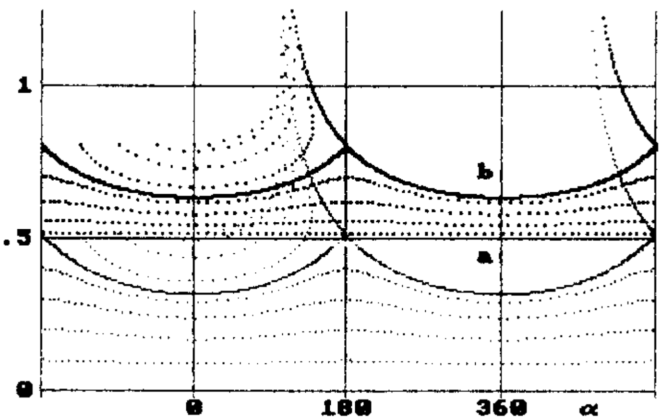
$da/dN = (\delta_\alpha/4)\ 3a^2b^2 \sin\alpha$
$db/dN = (\delta_\alpha/4)\ 2a^3b \sin\alpha$
$d\alpha/dN = (\delta_\alpha/4)((9ab^2 + 4a^3)\cos\alpha + 4)$

$2a^2 - 3b^2 = F$ fam.
$9ab^2 + 4a^3 = 4$ fxd. pt.
$(3/2)\ a^3b^2 \cos\alpha + a^2 + b^2 =$ const.



3,2 res. (dec.)

The straight line is $F = 0$ (slope $\sqrt{(3/2)}$). The fixed-point line is the outer one from $a = 1$, the inner one is the *adiabatic* limit.



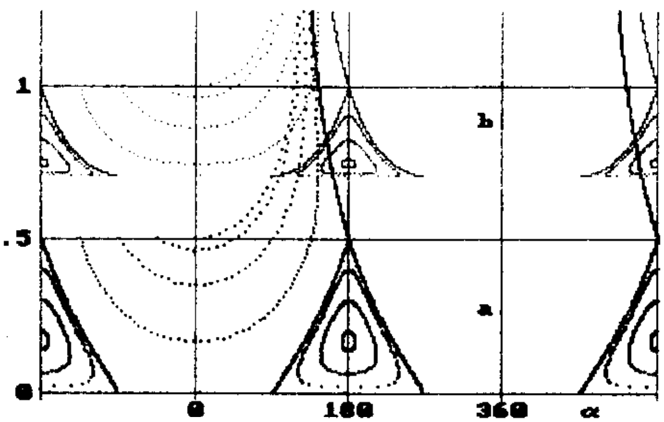
Trajectories $F = .5$ $(a=.5,\ b=0$

The darker points are for a, lighter for b. The fixed point is at the cusp at 180°. The separatrix is solid, and includes an upward branch. The incoming branch has been omitted for clarity. Beyond the separatrix α reverses and particles leave near 90°. Small amplitudes ($a\sim.5$) are almost unaffected.

Consider a stream of particles, of this family, moving just below the separatrix. If one slowly tunes away from resonance they will settle to constant amplitudes, still on the family line, preserving the phase volume. These are the adiabatic amplitudes plotted above and are the best measure of the stabitlity limit. They are found from the average of a^2b^2 along the separatrix. The line is plotted from such computations for many families.

Note that the usual pair of plots x-x', y-y' would not give any indication of the nature of the resonance because φ and ϑ continue forward without any apparent change even when α reverses direction.

We now turn to the sextupole driven $\nu_x + 2\nu_y$ resonance which has a new quirk of interest. The $a - b$ plot is on the opposite page, and one sees that it is an ellipse from 0,0 to .5,1 to 1,0. Family lines with F *negative*, that is above the straight line $F = 0$, will have <u>two</u> fixed points. The upper point is normal and locates the separatrix, the lower point near $a = 0$ does not have any connected trajectory but lies inside a special "locked" region (not really a coupling because a and b go up and down together but the effect is much the same). The darker part of the fixed point line terminates at $F = -.5$, which is the family line from 0,.707. The special property of this family is shown in the figure below.



$F = -.5 \quad (a=0,\ b=.7)$

For this family all particles are unstable or locked in the island, which extends from $a = 0$ to the fixed point. On adiabatic tuning particles do <u>not</u> enter the island so this family is a stability limit. The phase volume of the islands are subtracted from separatrix volume when computing the adiabatic stability line.

During a faster tuning some particles do enter and leave the island which modifies the amplitudes. There are however very few particles with small a and modest b, as pointed out above, and the emittance dilution is in a region where it doesn't much matter. The primary nuisance from the islands is the distortion of the trajectories moving around them.

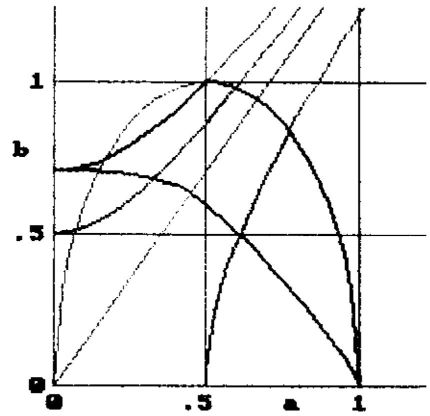
Any resonance with $m = 1$ will have islands near the *b-axis*, and with $n = 1$ near the *a-axis*, so we will see something similar for $\nu_x + 4\nu_y$ and even for the octupole coupling resonances.

$\nu_x + 2\nu_y = \text{int.} + \delta\sqrt{5}$ sextupole

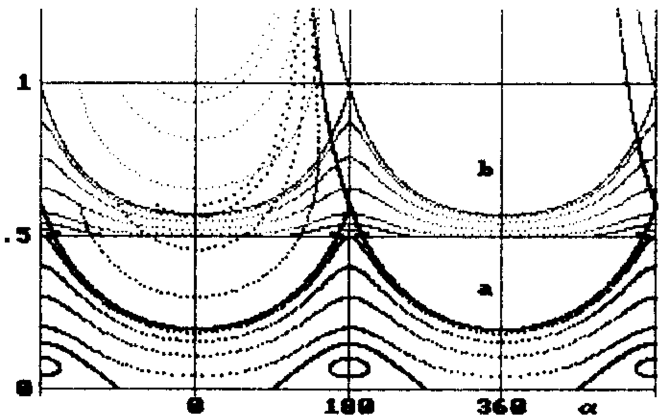
$$A = (\beta_0/B\rho)\sum(hv^2(Sl))_s \cos\alpha_s$$
$$a_0 = \delta_\alpha/A$$

$$da/dN = (\delta_\alpha/4)\ b^2 \sin\alpha$$
$$db/dN = (\delta_\alpha/4)\ 2ab \sin\alpha$$
$$d\alpha/dN = (\delta_\alpha/4)(b^2/a + 4a)\cos\alpha + 4)$$

$$2a^2 + b^2 = F \qquad \text{fam.}$$
$$b^2 + 4a^2 - 4a = 0 \qquad \text{fxd. pt.}$$
$$(3/2)\ ab^2 \cos\alpha + a^2 + b^2 = \text{const.}$$

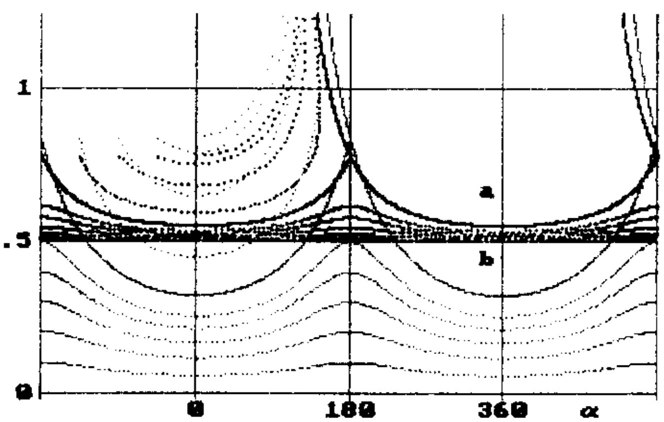


1,2 res. (sex.)



F = -.25 (b=.5, a=0)

There is a "locked" region near a = *0*, which is normally not populated. The principle effect is distortion of <u>all</u> small a motion.



F = .5 (a=.5, b=0)

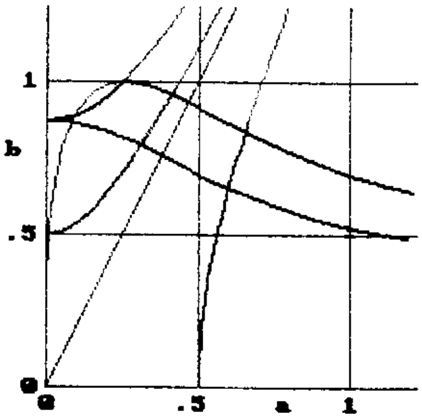
This side has no "island" and is smoother (for small *b*). One should compare this diagram to any of the decapole cases, which are much smoother. A higher dependence on amplitude would be very helpful.

$\nu_x + 4\nu_y$ = int. + $\delta\sqrt{17}$ decapole

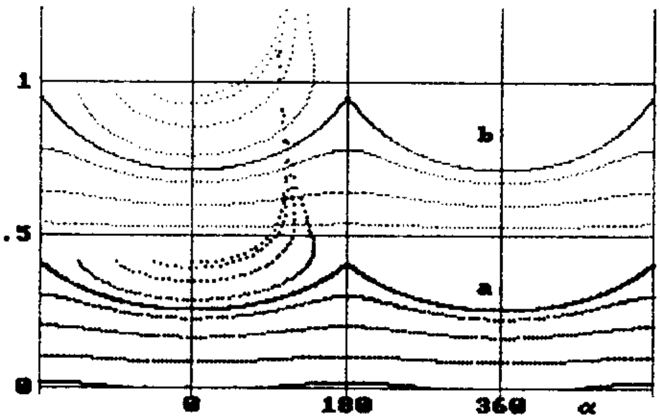
$a = (\beta_0/B\rho)\sum(hv^4\ (Dl))_s \cos\alpha_s$
$a_s^3 = 2\delta_\alpha/A$

$da/dN = (\delta_\alpha/8)\ b^4 \sin\alpha$
$db/dN = (\delta_\alpha/8)\ 4ab^3 \sin\alpha$
$d\alpha/dN = (\delta_\alpha/8)((b^4/a + 16ab^2)\cos\alpha + 8)$

$4a^2 + b^2 = F$ fam.
$b^4 + 16a^2b^2 - 8a = 0$ fxd. pt.
$(5/4)\ ab^4 \cos\alpha + a^2 + b^2 =$ const.

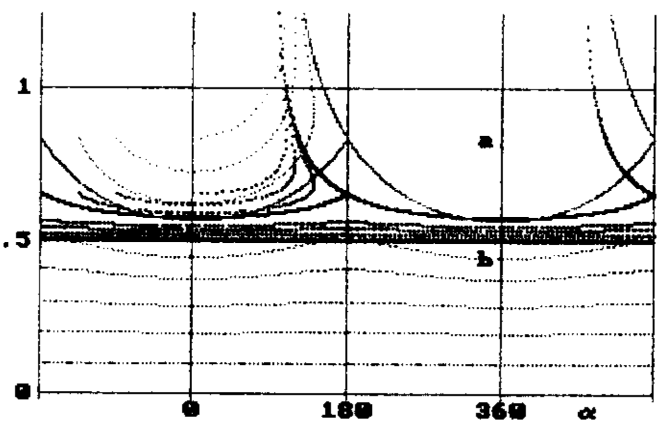


1,4 res (dec)



F = -.25 (b=.5, a=0)

This resonance also has a "locked" island near a = 0 but because of the amp³ dependence it has almost vanished at this *b*.



F = 1 (a=.5, b=0)

The flattest trajectories! Unfortunately the adiabatic limit is wide open in the a direction and there is no point using this resonance instead of one of the others.

Thirds resonances come quite naturally from decapoles for example in one dimension we find ourselves expanding

$$\cos^5 \varphi = (1/16)(\cos 5\varphi + 5\cos 3\varphi .....$$

and there it is, with a fat multiplier yet! Again to find one of our resonances, I used

$$B_x = D\ (...6x^2y^2 \qquad B_y = D\ (...4x^3y$$

expanding $\cos^3\varphi \cos^2\vartheta = (1/16)(\cos 3\varphi+2\vartheta + 3\cos \varphi+2\vartheta + ...$

and for the other resonance I used

$$B_x = D\ (...y^4 \qquad B_y = D\ (...4xy^3$$

expanding $\cos \varphi \cos^4\vartheta = (1/16)(\cos \varphi+4\vartheta + 4\cos \varphi+2\vartheta + ...$

and both contribute to $\nu_x + 2\nu_y$.

We now face four driving terms

$$A_{32}\ ..h^3v^2 \cos \alpha, \quad B_{32}\ ..h^3v^2 \sin \alpha, \quad A_{14}\ ..hv^4 \cos \alpha, \ B_{14}\ ...hv^4 \sin \alpha$$

which look familiar but now $\alpha = \varphi + 2\vartheta$. Only one term can be eliminated by shifting (B_{32}). I will simplify the problem by defining

$$a_0^3 = \delta_a/A_0, \quad A_{32} = 2/3\ A_0, \quad A_{14} = \tfrac{1}{2}A_0, \quad B_{14} = 0$$
$$\underline{or} \quad A_{14} = 0, \quad B_{14} = \tfrac{1}{2}A_0$$

The fractions are a friendly choice which happens to make the adiabatic separatrix approximately symmetric for a and *b*. The two examples are for driving terms "in phase" and "out of phase". Fortunately the diagrams are very similar.

The diagrams have "families" (and phase angles) like $\nu_x + 2\nu_y$, as they should, but all other lines are just like a superposition of the two previous decapole resonances, with $3\nu_x + 2\nu_y$ dominating at low *b* and $\nu_x + 4\nu_y$ at low a, which is great. This is even more apparent for the out-of-phase case.

In the out-of-phase case we can no longer use $\cos \alpha = -1$ for the fixed points. In fact one must choose $\varphi = \tan^{-1}(-b^2/2a^2)$ which keeps shifting along the line. Actually one must be in the correct quadrant so

$$\sin \varphi = -2a^2/(4a^2 + b^4)^{1/2}, \quad \cos \varphi = b^2/(4a^4 + b^4)^{1/2} \qquad \text{fxd. pts.}$$

This kind of phase-shifting is endemic, when a dominates (and therefore A_{32}) then the phase is as usual, but when *b* dominates features are shifted 90° earlier because B_{14} dominates. This causes only one minor operational change: the asymptotic exit phase is 45° instead of 90°.

$\nu_x + 2\nu_y$ = int. + $\delta\sqrt{5}$ decapole, in-phase case

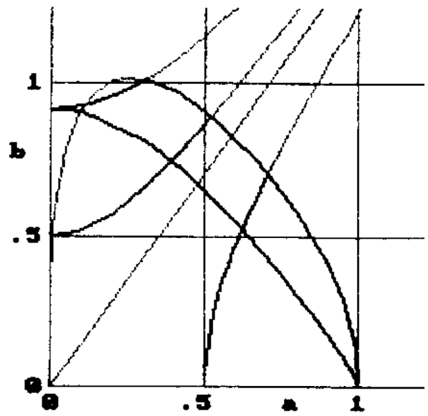
$$A_{32} = 2/3\, A_0 = (\beta_0/B\rho)\sum(h^3v^2(Dl))_s \cos\alpha_s$$
$$A_{14} = 1/2\, A_0 = (\beta_0/B\rho)\sum(hv^4(Dl))_s \cos\alpha_s$$
$$a_0^3 = \delta_\alpha/A_0$$

$$da/dN = (\delta_\alpha/8)(2a^2b^2 + b^4)\sin\alpha$$
$$db/dN = (\delta_\alpha/8)(4a^3b + 2ab^3)\sin\alpha$$
$$d\alpha/dN = (\delta_\alpha/8)((8a^3 + 14ab^2 + b^4/a)\cos\alpha + 8)$$

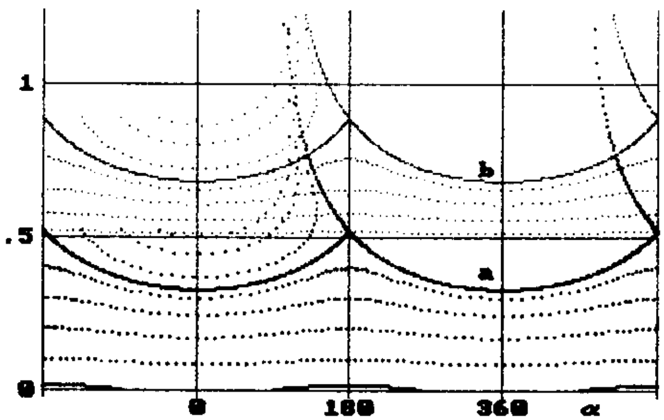
$2a^2 - b^2 = F$ fam.

$8a^4 + 14a^2b^2 + b^4 - 8a = 0$ fxd. pt.

$(3/4)(2a^3b^2 + ab^4)\cos\alpha + a^2 + b^2 = \text{const.}$

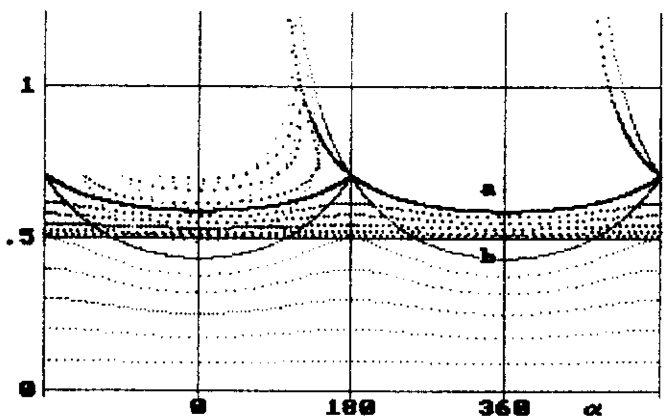


in-phase



$F = -.25$ $(b=.5,\ a=0)$

On the $a - b$ plot one sees a very good shape for the adiabatic stability limit. This plot shows a small "island" and flat trajectories similar to the *1, 4* resonance. The fixed point amplitudes are different primarily because the family lines have a different shape.



$F = .5$ $(a=.5,\ b=0)$

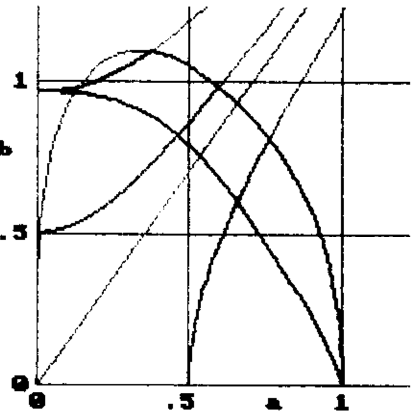
On this side the trajectories resemble the *3, 2* decapole resonance. All in all an excellent resonance for scraping.

$\nu_x + 2\nu_y$ = int. + $\delta\sqrt{5}$ decapole, out-of-phase case

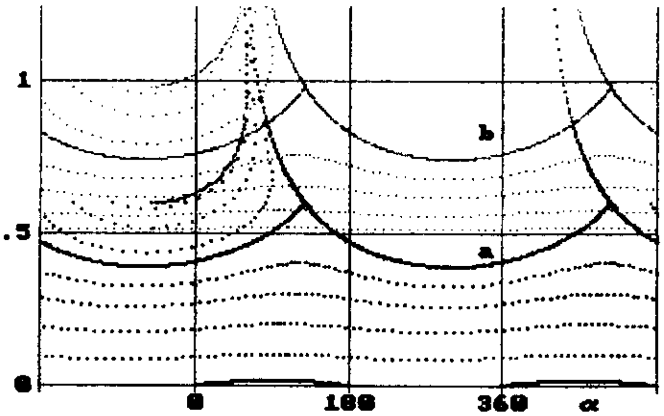
$A_{14} = 2/3\ A_0 = (\beta_0/B\rho)\sum(h^3v^2(Dl))_s \cos\alpha$
$B_{14} = 1/2\ A_0 = (\beta_0/B\rho)\sum(hv^4(Dl))_s \sin\alpha$
$a_0^3 = \delta_\alpha/A_0$

$da/dN = (\delta_\alpha/8)(2a^2b^2 \sin\alpha\ + b^4 \cos\alpha)$
$db/dN = (\delta_\alpha/8)(4a^3b \sin\alpha\ + 2ab^3 \cos\alpha)$
$d\alpha/dN = (\delta_\alpha/8)((8a^3 + 6ab^2) \cos\alpha\ - (b^4/a + 8ab^2) \sin\alpha)$

$2a^2 - b^2 = F$ fam.
$16a^6 + 12a^4b^2 + 8a^2b^4 + b^6 - 8a(4a^4 + b^4)^{1/2} = 0$ f.p.
$(3/4)(2a^3b^2 \cos\alpha\ - ab^4 \sin\alpha) + a^2 + b^2 = \text{const.}$

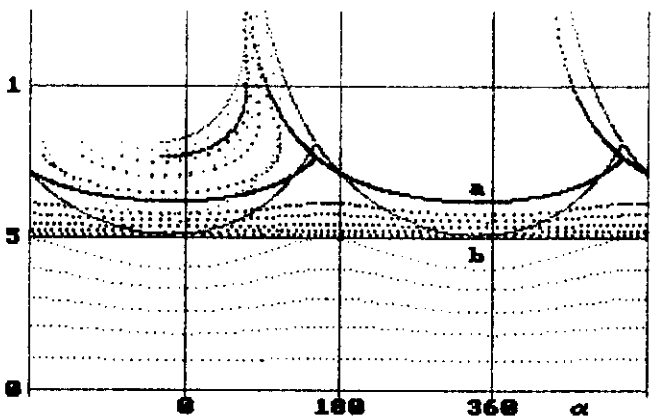


out-of-phase drivers



$F = -.25$ $(b=.5,\ a=0)$

The important point is the similarity of the a - *b* plots for this case and the in-phase case on the previous page. The trajectories show some interesting phase shifts from B_{14} instead of A_{14}.



$F = .5$ $(a=.5,\ b=0)$

Note that the exit *α* is now approaching 45° instead of the usual 90°. Otherwise the phase shift has no effect on performance, but it does complicate the arithmetic.

On Coming Out

The magnitude of the driving term determines the step spacing for exitting protons. It must be large enough to make clean hits on the target but not so large that protons just missing are lost before they again have maximum displacement at the target. Once the driving term is chosen the *tuning* sets the amplitude level for the separatrix. We can make good estimates of these quantities by assuming that $\sin\alpha = 1$, $F = 0$, and using the differential equations. In fact $\cos\alpha \to 0$ on exit because it is multiplied by a higher power of the amplitudes in the constant-of-motion expressions; also the families lines converge on the $F = 0$ straight line.

Let me assume for purposes of comparison that the target is a distance $r = (a^2 + b^2)^{1/2}$ = 1.2 cm. from the center along the $F = 0$ line, that a proton just missing returns to the target in 10 turns (tunes near .4, .3), and that it should then be at r = 1.7 cm. I also assume that a_0 = .6 cm (1 on our diagrams) for the separatrix. In these expressions dimensions are in cm. but are still in beta space:

$\nu_x + 2\nu_y$, sextupole

$da/dN = (A/4)\, b^2 \sin\alpha$	
$da/dN = (A/2)\, a^2$	$(\sin\alpha = 1,\ F = 0)$
$dr/dN = (a/2)\, r^2/\sqrt{3}$	$(r = a\sqrt{3})$
$(1/r_0) - (1/r_N) = N A/2\sqrt{3}$	
$A = .085/\text{cm.}$	(1.2 cm., 1.7 cm., 10 turns)
$2\pi\, \delta\sqrt{5} = A\, a_0$	(from before)
$\delta = .0036$	(a_0 = .6 cm.)

These are comfortable values. The driving term A can be implemented using correction style elements.

$3\nu_x + 2\nu_y$, decapole

$da/dN = (A/8)\, 3a^2b^2 \sin\alpha$	
$da/dN = (A/4)\, a^4$	$(\sin\alpha = 1,\ F = 0)$
$dr/dN = (A/4)\, (3/5)^{1.5}\, r^4$	$(r = a\sqrt{(5/3)})$
$(1/r_0^3) - (1/r_N^3) = .349\, A\, N$	
$A = .103/\text{cm}^3$	(1.2 cm., 1.7 cm., 10 turns)
$2\pi\, \delta\sqrt{13} = A\, a_0^3/2$	
$\delta = .0005$	(a_0 = .6 cm.)

Not comfortable. decapoles are $\sim$20 times harder to build.

$\nu_x + 2\nu_y$, **decapoles** (in-phase case)

$$da/dN = (A_0/8)(2a^2b^2 + b^4)\sin\alpha$$

$$da/dN = A_0\, a^4 \qquad (\sin\alpha = 1,\ F = 0)$$

$$dr/dN = A_0\, r^4/3\sqrt{3} \qquad (r = a\sqrt{3})$$

$$(1/r_0^3) - (1/r_N^3) = N\, A_0/\sqrt{3}$$

$$A_0 = .065/cm^3$$

$$A_{32} = .044/cm^3,\ A_{14} = .033/cm^3 \qquad (1.2\text{ cm., } 1.7\text{ cm., } 10\text{ turns})$$

$$2\pi\,\delta\sqrt{5} = A_0\, a_0^3$$

$$\delta = .0010 \qquad (a_0 = .6\text{ cm.})$$

The out-of-phase case gives the same result. We have gained a factor of 2½ compared to the usual decapole resonance above, and we need it.

The following is an example of an arrangement of decapoles which produces the driving terms for this resonance. I assume 60° cells:

	F	D	F	D	F
(Dl)	-1	-.6	1	.6	-1
$\varphi + 2\vartheta$	-180	-90	0	90	180
h^3v^2	1/3	$1/3\sqrt{3}$	1/3	$1/3\sqrt{3}$	1/3
hv^4	1/9	$1/\sqrt{3}$	1/9	$1/\sqrt{3}$	1/9

$$A_{32} = 1 \quad B_{32} = .4/\sqrt{3}, \quad \rightarrow 1.03 \text{ @ } 13^\circ$$

$$A_{14} = 1/3 \quad B_{14} = 1.2/\sqrt{3}, \quad \rightarrow 0.77 \text{ @ } 64^\circ$$

The ratio is correct (accuracy not required). Our "observation point" will be 13° downstream (in α) from the central quadrupole. The unit decapole will be (Dl) = 1.3 T/cm³, or 100 cm. long and .013 T/cm⁴. At a 4 cm. radius the field would be 3.3 T, which practical but is not a "correction" element.

It is interesting to note that the driving terms from this array for the regular resonances 5,0 3,2 1,4 are .0017, .005, and .015/cm³. The small value for the $5\nu_x$ resonance is important if our operating point is near .4,.3 . A proper design would be much more sophisticated (see ***Distortion Functions*** for a design procedure for avoiding non-resonant effects).

With an exit angle $\alpha = 90^\circ$, significant combinations are

$$\varphi = 30 \qquad \vartheta = 30, \text{ or } 210^\circ$$

$$\varphi = 150 \qquad \vartheta = 330, \text{ or } 150^\circ$$

$$\varphi = 270 \qquad \vartheta = 270, \text{ or } 90^\circ$$

because a target 150, 30, or 90° downstream (ϑ and φ) will be at maximum displacement in x and y.

Comments and Conclusions

Decapole driven resonances are hard to implement, and unless there is some serious tuning problem one would only consider the decapole $\nu_x + 2\nu_y$ resonance. The advantage of decapoles is the undisturbed beam permitting continuous scraping.

The sextupole driven resonance is relatively easy to implement and would probably work very nicely for intermittent scraping. One wonders whether the beam distortion might be tolerable for continuous scraping. Possible problems can arise at four different levels.

The first problem could be that a single beam does not work well close to this resonance, with or without the scraping turned on. One problem to remember is that second-order tune-shift from sextupoles can be enhanced by the first order distortion. If problems arise from random field errors then it is possible to "clean-up" the particular tune area, however if they arise from systematic elements there may be a conflict which cannot be resolved. One should remember that systematic effects will be very different when the low-beta sections are turned on and the ring loses all symmetry (in phase space).

The simplest of the beam-beam effects is that the distortion degrades the beam density. I estimate that when using the 6 mm level of scraping the decrease is small, and would be compensated by less than 1% decrease in β^*.

It is possible that the beam-beam effect finds this particular tune disturbing (without scraping on) or, conversly, the scraping resonance finds the peculiar tune-shift curve from beam-beam interaction disturbing. Both of these problems could more easily occur if one tunes to the wrong side of the resonance where the beam-beam tune-shift, which is large only for small amplitudes, is towards the resonance. For $\bar{P}$-P one should tune on the high side.

Finally it is possible that the multipole field from the scraper driver combines with the non-multipole field of beam-beam interaction to create strange effects. This is clearly an effect that cannot be analyzed, which is why it is a popular explanation for beam-beam problems when analysis fails to find any problem with the beam-beam interaction itself, as it does for protons. It may even be real, in which case a failed scraper experiment would be a great success.

My suspicion is that, with a little preliminary prophylaxis, none of the above will occur.

There are some obvious limitations to the type of analysis in this paper for the exitting particles. The basic assumption that non-resoant terms will average out in a few turns is no longer meaningful. Driving arrays that locally cancel non-resonant distortion, such as *cos* φ compenents when creating *cos* $\varphi+2\vartheta$ components, will produce trajectories much like the above. Tracking studies are needed for precise design of the actual trajectory. It may be possible to split the drivers on either side of the target and to gain an advantage from a local non-cancellation of the other terms.

Conversion from beta space to real space provides an opportunity to use any higher betas that are available. The above analysis was for arrays of *normal* elements which emphasizes vertical displacement at the target. *Skew* components work just as well and reverse the role of a and b and also β_x and β_y.

There are probably better ways to tackle resonant scraping. There is much room for invention and design. The hope is that this paper will stimulate both theoretical and practical interest. I am sure we all agree that the dipoles should no longer be permitted to dictate where protons are lost, particularly in a superconducting ring with an excellent aperture, and with the forefront experiments.